Signal detection

Read:
• chapter 5
•E.L. Hahn. Spin Echoes. Phys. Rev., 80:580, 1950.

Signal Reception in MRI

- MR signal: voltage induced in the RF coil by changes in magnetic flux from the precessing magnetization in the our sample.

$$emf = -\frac{d\Phi}{dt}$$

where Φ is the flux in the coil.

Signal Equation

$$s(t) = \int_x \int_y m(x,y) e^{-i2\pi[k_x(t)x + k_y(t)y]} dxdy$$

where

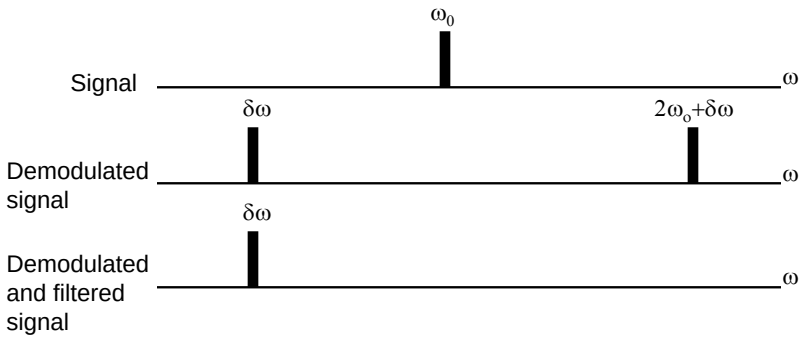
$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$$

Signal Demodulation

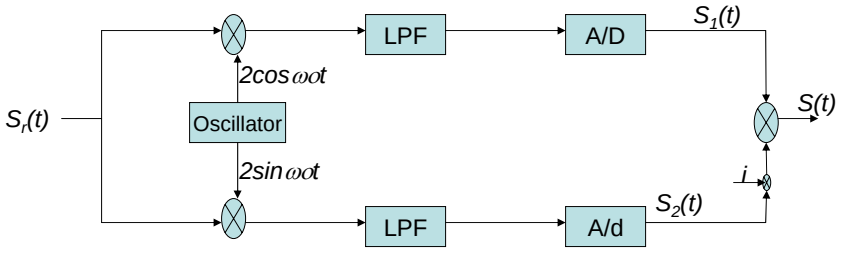
- Demodulation corresponds to the multiplication of the signal by a sinusoid or cosinusoid with a frequency near or at ω_0.
- The high-frequency (MHz) is demodulated in a part of the receiver (demodulator) and converted to a low frequency (kHz) signal that contains the "modulated information", i.e. the frequency range across the field of view encoded by the frequency encoding gradient.
- Both sine and cosine multiplication are considered corresponding to data storage in two channels respectively, real and imaginary.
- Low pass filtering applied to the demodulated signal eliminates the high frequency components.

Demodulated Signal



The shift in frequency for signals from a uniform sample due to demodulation and filtering

Signal Demodulation



Signal Demodulation

The upper channel

$$s_1(t) = LPF\{2m_0\cos(\omega_0 t + \Delta\omega t)\cos(\omega_0 t)\}$$

$$= LPF\{m_0[\cos(\Delta\omega t) + \cos(2\omega_0 t + \Delta\omega t)]\}$$

$$= m_0\cos(\Delta\omega t)$$

The lower channel

$$s_2(t) = LPF\{2m_0\cos(\omega_0 t + \Delta\omega t)\sin(\omega_0 t)\}$$

The negative has been chosen as a convention

$$= LPF\{m_0[-\sin(\Delta\omega t) + \sin(2\omega_0 t + \Delta\omega t)]\}$$

$$= -m_0\sin(\Delta\omega t)$$

The combined signal

$$s(t) = s_1(t) + is_2(t)$$

$$= m_0\exp(-i\Delta\omega t)$$

$$= m_0\exp(-i\Delta\omega t)$$

$$\sin s + \sin t = 2\sin\frac{s+t}{2}\cos\frac{s-t}{2}$$

$$\sin s - \sin t = 2\cos\frac{s+t}{2}\sin\frac{s-t}{2}$$

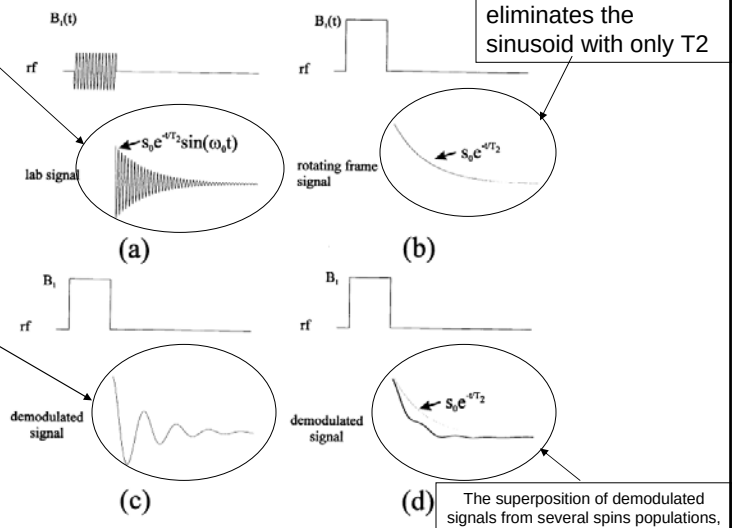
$$\cos s + \cos t = 2\cos\frac{s+t}{2}\cos\frac{s-t}{2}$$

$$\cos s - \cos t = -2\sin\frac{s+t}{2}\sin\frac{s-t}{2}$$

Free Induction Decay and T_2^*

No Demodulation is applied, rapid oscillations at frequency ω are damped by T2

Demodulation On resonance eliminates the sinusoid with only T2



Demodulation Slightly off resonance leaves a low frequency components

The superposition of demodulated signals from several spins populations, each with different ω_0, leads to a signal that decay that decays faster than a single demodulated signal which only has T2 damping

- FID: free – meaning it is not being driven by an RF pulse, induction – the action of a magnetic moment precessing around a magnetic field, and decay – meaning T2 decay).

Variations of the Magnetic Field

- There many things that can affect the magnetic field. These include
  - Magnetic field inhomogeneity – this reflects our inability to make the field perfectly homogeneous.
  - Magnetic susceptibility – this is the magnetization of tissue itself. Different tissues, bones and the surrounding air all have magnetic susceptibility differences of several ppm. The net field is given as $B = B0(1+\chi)$, where χ is the magnetic susceptibility (χ_{air} is nearly 0, χ_{water} is about –9x10-6 or –9 ppm).

Variations of the Magnetic Field (cont'd)

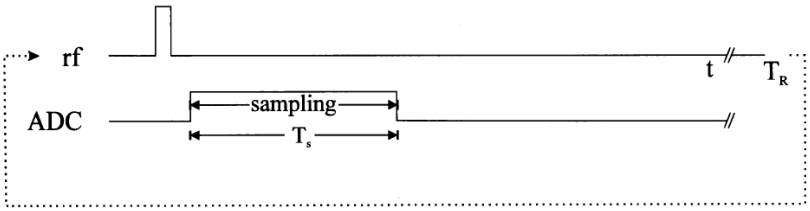
– Chemical shift –different shielding of the nucleus from the surrounding electron clouds. The net field is $B = B0(1-\sigma)$, where σ is the chemical shift (a positive chemical shift implies shielding of the nucleus or a downward shift in the field). A common chemical shift the shift between water protons (bonded to O) and fat protons (bonded to C): σ_{wf} is about 3.5 ppm.

Phase Behavior and Phase Conventions

- The contributions of the phase due to the magnetization (spin isochromat) is:

$$\begin{aligned}\phi(\vec{r},t) &= -\omega(\vec{r})t + \phi_0(\vec{r}) \\ &= -\gamma B_z(\vec{r})t + \phi_0(\vec{r})\end{aligned}$$

FID Sequence Diagram and Sampling

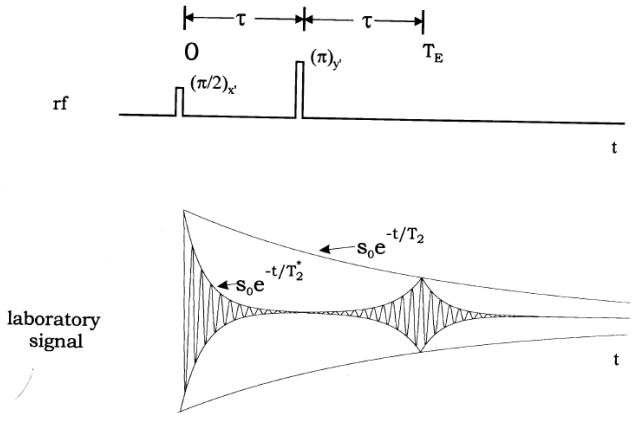


The continuous *emf* is measured by the A/D. The total sampling time is

$$T_s = (N-1)\Delta t$$

Q: what is the number and frequency of the discrete steps

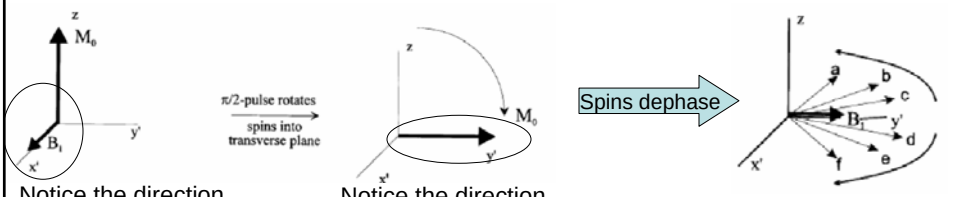
Spin Echo Method



Steps for Spin Echo

1. The magnetization is tipped by the first pulse immediately in the transverse plane. The spins at different positions begin to dephase. The accumulated phase in the rotating frame is

$$\phi(\vec{r},t) = -\gamma \Delta B\ (\vec{r})t \qquad 0 < t < \tau$$



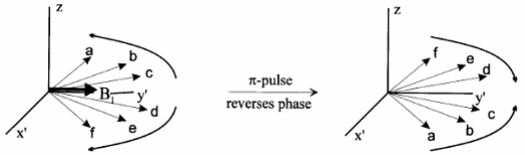
Notice the direction of the RF pulse

Notice the direction of the magnetization

Steps for Spin Echo (cont'd)

2. Another RF pulse with twice the amplitude is applied flipping the sign of the phase

$$\phi(\vec{r},\tau^{+}) = \gamma \Delta B\ (\vec{r})t$$



Steps for Spin Echo (cont'd)

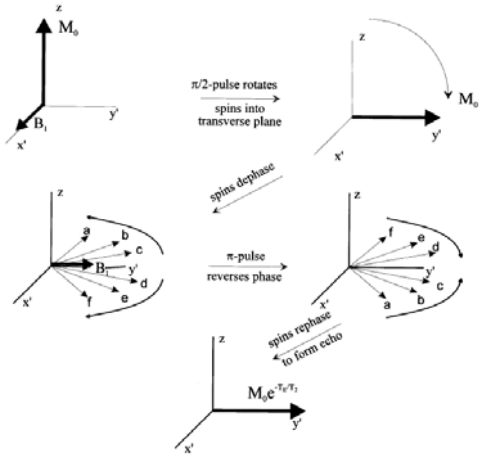
3. The spins continue to accumulate phase according to

$$\begin{aligned}\phi(\vec{r},t) &= \phi(\vec{r},\tau^{+}) - \gamma\Delta B\ (\vec{r})(t-\tau) \\ &= -\gamma\Delta B\ (\vec{r})(t-2\tau) \\ &= -\gamma\Delta B\ (\vec{r})(t-T_E)\end{aligned}$$

With TE = 2τ

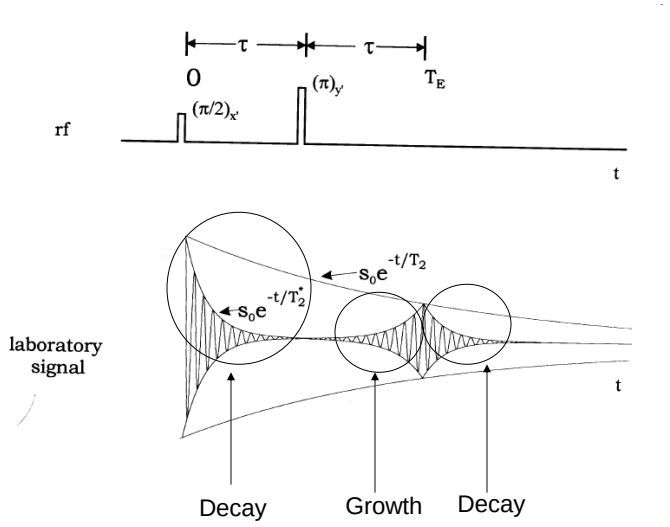
•Since the rate at which phase is accumulated by each spin is unchanged, all of the spins will return to the initial phase value at the echo time
•All of the spins will be aligned at T_E, and the realignment of the spins is called a spin echo

Spin Echo



Q: What is the benefit of a spin echo

Spin Echo

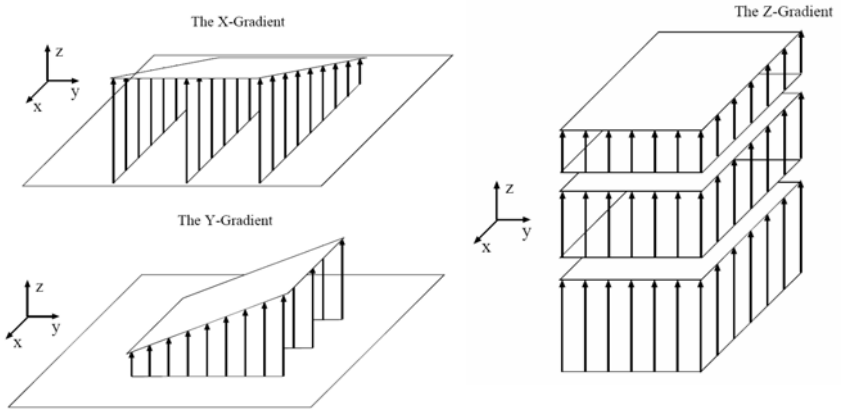


Limitations of the Spin Echo

- The signal peaks at the echo, but it recovers only to the level dictated by the T_2 envelope.
- The inhomogeneities do not stay fixed in time, the rates at which phase is accumulated change with time (no refocusing is possible 100%)

Gradients

- Purpose of gradient fields: spatial localization.
- **Is this there only job ?**



HW # 4: 1D Fourier Imaging

- Let the variation be along the *x* direction, $\Delta B(x, y, t) = G_x \cdot x$, find:
  - The solution to the bloch equation
  - The signal equation
  - Solve the signal equation and find the magnetization if $m(x) = \mathrm{rect}(x/X)$